Descarte's Rule of Signs

Descarte's Rule of Signs – Positive Roots :→

No equation can have more positive roots than it has changes of sign from + to –, and from – to +, in the terms of first member.

Negative Roots :→

No equation can have a greater number of negative roots than there are changes of sign in terms of the polynomial $f(-x)$.

Verification (Proof) :→

Let the signs of a polynomial be

$$+ \; + \; - \; + \; - \; + \; - \; -$$

The given polynomial has five changes. Now we multiply the given polynomial by a factor $(x-h)$ corresponding to the positive root h.

sign of this binomial is $+ \; -$

$$
\begin{array}{ccccccccc}
+ & + & - & + & - & + & - & - & \\
+ & - & & & & & & & \\
\hline
+ & + & - & + & - & + & - & - & \\
& - & - & + & - & + & - & + & + \\
\hline
+ & \pm & - & + & - & + & - & \mp & +
\end{array}
$$

The resultant polynomial has two ambiguous sign and we write

'2^p' different ways, $p =$ number of ambiguous signs

$$
\begin{array}{ccccccccc}
+ & + & - & + & - & + & - & + & + \\
+ & + & - & + & - & + & - & - & + \\
+ & - & - & + & - & + & - & + & + \\
+ & - & - & + & - & + & - & - & +
\end{array}
$$

(They having six changes of sign)

In all possible ways the resulting polynomial has six changes of sign i,e; at least one more than the number of changes of signs in the orignal polynomial.

Hence it is clear that corresponding to the introduction of a positive root the resulting polynomial has at least one more change of sign. Now if $\phi(x)$ be the product of factors corresponding to -ve & complex roots and $\alpha, \beta, \gamma, \ldots$ be the positive roots $(x-\alpha)(x-\beta)(x-\gamma) \ldots\ldots$

Then each multiplication will introduce one more than change of sign. Hence the number of positive roots cannot exceed the number of sign of $f(x)=0$,

<u>Negative roots</u> :→

We know that negative roots of $f(x)=0$, are positive roots of $f(-x)=0$ & as such the number of -ve roots of $f(x)=0$ cannot exceed the number of changes of signs in $f(-x)=0$.